

# Why do we use radians?

This document explains why we usually use radians in preference to degrees when we're working with calculus.

You've seen that, by convention, the standard functions  $f(x) = \sin x$ ,  $f(x) = \cos x$  and  $f(x) = \tan x$  are the trigonometric functions sine, cosine and tangent with the input variable  $x$  measured in radians.

The corresponding functions, with the the input variable measured in degrees, are given by the formulas  $g(x) = \sin x^\circ$ ,  $g(x) = \cos x^\circ$  and  $g(x) = \tan x^\circ$ .

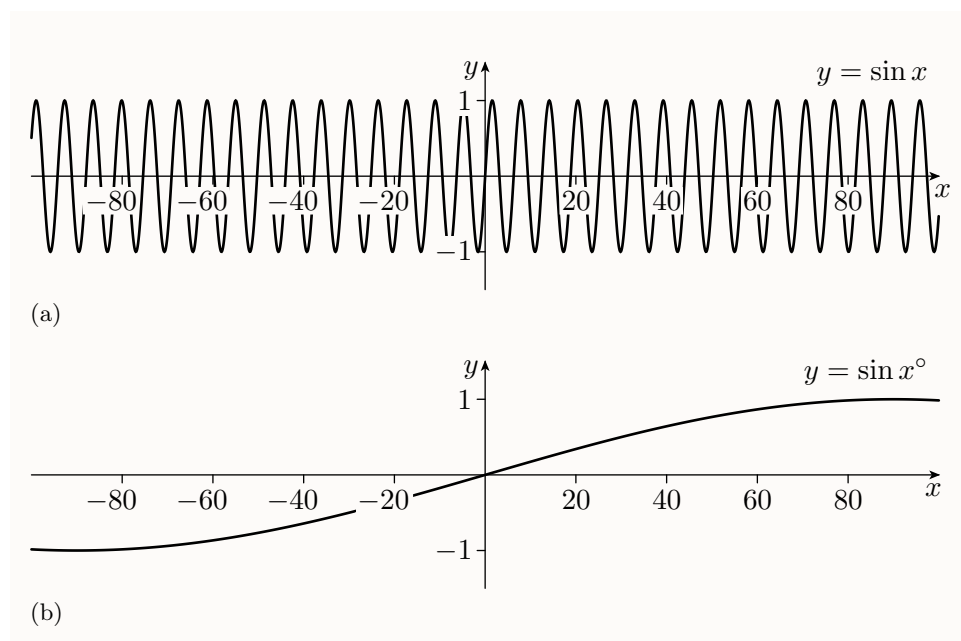
For example, if  $f(x) = \sin x$  and  $g(x) = \sin x^\circ$ , then

$$f(2) = \sin 2 = 0.909 \text{ (to 3 d.p.)},$$

whereas

$$g(2) = \sin 2^\circ = 0.035 \text{ (to 3 d.p.)}.$$

Let's consider the two functions  $f(x) = \sin x$  and  $g(x) = \sin x^\circ$  in more detail. Their graphs have the same basic shape, but the graph of the second function is much more stretched out in the  $x$ -direction. This is illustrated in Figure 1, which shows the two graphs for values of  $x$  between  $-100$  and  $100$ .



**Figure 1** The graphs of (a)  $y = \sin x$  (b)  $y = \sin x^\circ$

You can see that when  $x = 0$  the gradient of the first graph is much larger than the gradient of the second graph, and similarly the gradients of the two graphs are different at other values of  $x$ . So the two functions will have different formulas for their derivatives.

You've seen that the formula for the derivative of the function  $f(x) = \sin x$  is simply

$$f'(x) = \cos x.$$

You can work out the formula for the derivative of the function  $g(x) = \sin x^\circ$  by using the formula above together with the rule for differentiating a function of a linear expression, as follows. Since

$$x^\circ = \frac{\pi}{180} x \text{ radians,}$$

we have

$$\begin{aligned} g'(x) &= \frac{d}{dx} (\sin x^\circ) \\ &= \frac{d}{dx} \left( \sin \left( \frac{\pi}{180} x \right) \right) \\ &= \frac{\pi}{180} \cos \left( \frac{\pi}{180} x \right) \\ &= \frac{\pi}{180} \cos x^\circ. \end{aligned}$$

So the formula for the derivative of the function  $g(x) = \sin x^\circ$  contains the inconvenient coefficient  $\pi/180$ .

You can work out in the same way that the formulas for the derivatives of the functions  $g(x) = \cos x^\circ$  and  $g(x) = \tan x^\circ$  also contain the same inconvenient coefficient.

So the formulas for the derivatives of the functions  $f(x) = \sin x$ ,  $f(x) = \cos x$  and  $f(x) = \tan x$  are simpler than the formulas for the derivatives of the functions  $g(x) = \sin x^\circ$ ,  $g(x) = \cos x^\circ$  and  $g(x) = \tan x^\circ$ . This is the reason why we usually work with the first three functions here, rather than the latter three.

There are of course many other possible units that you could use to measure angles. For example, rather than saying that there are 360 units in a full turn (this gives degrees), or  $2\pi$  units in a full turn (this gives radians), you could say that there are 100 units in a full turn, or  $27\frac{5}{8}$  units, or  $\sqrt{2}$  units, and so on. Different units give different amounts of squashing or stretching of the graphs of the trigonometric functions in the  $x$ -direction, and hence different coefficients in the formulas for their derivatives. The units that give a coefficient of exactly 1 are radians, and this is the reason why we use them. In fact, radians give the amount of squashing or stretching that's needed to ensure that the gradient of the sine function at  $x = 0$  is exactly 1.